Introduction to Factor Analysis for Marketing

SKIM/Sawtooth Software Conference 2016, Rome

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Special thanks to Josh Lewandowski at Google for helpful feedback (errors are mine!)

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Introduction

Overview

This presentation is primarily a *conceptual* introduction to factor analysis. We will focus more on concepts and best practices than on technical details.

- Motivation & Basics
- Exploratory Factor Analysis (EFA)
- Confirmatory Factor Analysis (CFA)
- Further Learning

Examples are shown in R, but the process and results are similar in SPSS and other statistics packages.

Motivation: Example

Consider a standard product or satisfaction survey. These often have multiple items for different aspects of customers' concerns. For example, the author's "Product Interest and Engagement Scale" has 11 items including:

- I never think about ___
- I am very interested in ___
- In choosing a ___ I would look for some specific features or options.
- Some ___ are clearly better than others.
- When people see someone's ___ , they form an opinion of that person.
- A ___ expresses a lot about the person who owns it.
- It is important to choose a ___ that matches one's image.

Do these express different **factors** of interest?

Exploratory Factor Analysis

The General Concept (version 1)

From the original variables, factor analysis (FA) tries to find a smaller number of derived variables (*factors*) that meet these conditions:

1. Maximally capture the correlations among the original variables (after accounting for error)
2. Each factor is associated clearly with a subset of the variables
3. Each variable is associated clearly with (ideally) only one factor
4. The factors are maximally differentiated from one another

These are rarely met perfectly in practice, but when they are approximated, the solution is close to "simple structure" that is very interpretable.

The General Concept, version 1 example

Consider (fictonal) factor analysis of a standardized school test:

Variable	Factor 1	Factor 2	Factor 3
Arithmetic score	0.45	**0.88**	0.25
Algebra score	0.51	**0.82**	0.03
Logic score	0.41	0.50	0.11
Puzzle score	0.25	0.42	0.07
Vocabulary score	0.43	0.09	**0.93**
Reading score	0.50	0.14	**0.85**

We might interpret this as showing:

- Factor 1: general aptitude
- Factor 2: mathematical skills
- Factor 3: language skills

The General Concept in Simplified Math

In short: the factor loading matrix, times itself (its own transpose), closely recreates the variable-variable covariance matrix.

$$LL'(+E) = C$$

$$LoadingsLoadings'(+Error) = Covariance$$

$$\begin{bmatrix} F1.v1 & F2.v1 \\ F1.v2 & F2.v2 \\ F1.v3 & F2.v3 \end{bmatrix} \begin{bmatrix} F1.v1 & F1.v2 & F1.v3 \\ F1.v2 & F2.v2 & F2.v3 \end{bmatrix} = \begin{bmatrix} v1.v1 & v1.v2 & v1.v3 \\ v2.v1 & v2.v2 & v2.v3 \\ v3.v1 & v3.v2 & v3.v3 \end{bmatrix}$$

The General Concept (version 2)

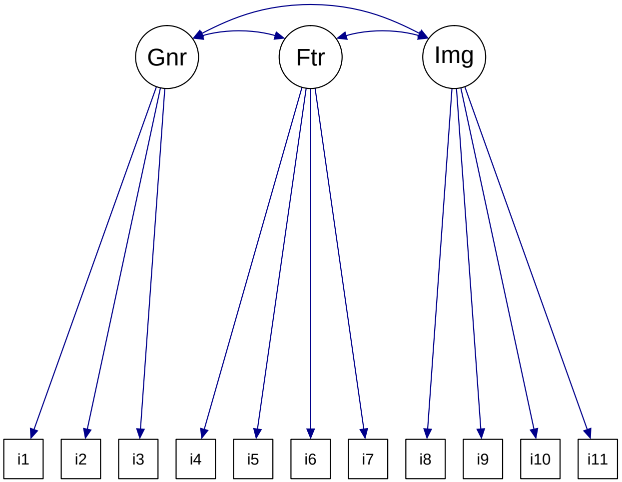
Another way to look at FA is that it seeks *latent variables*. A latent variable is an unobservable data generating process — such as a mental state — that is manifested in measurable quantities (such as survey items).

The product interest survey was designed to assess three latent variables:

- General interest in a product category
- Detailed interest in specific features
- Interest in the product as an "image" product

Each of those is assessed with multiple items because any single item is imperfect.

Visual Example for Product Interest



Very Different Modes of Factor Analysis

Exploratory Factor Analysis (EFA)

- Asks what the factors *are* in observed data
- Requires interpretation of usefulness
- Before assuming it's correct, confirm with CFA

Confirmatory Factor Analysis (CFA)

- Asks how well a proposed model *fits* given data
- Is a flavor of structural equation modeling (SEM)
- Doesn't give an absolute answer; should compare models

Key terms and symbols

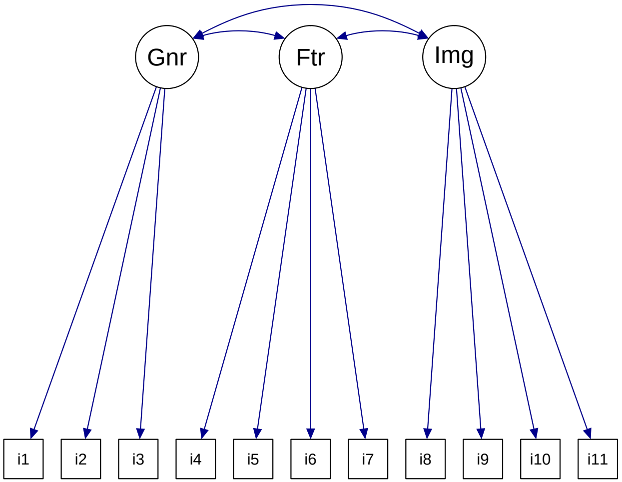
Latent variable: a presumed cognitive or data generating process that leads to observable data. This is often a theoretical *construct*. Example: *Product interest*. Symbol: circle/oval, such as (F1).

Manifest variable: the observed data that expresses latent variable(s). Example: "How interested are you in this product? [1-5]" Symbol: box, such as [Item1].

Factor: a dimensional reduction that estimates a latent variable and its relationship to manifest variables. Example: InterestFactor.

Loading: the strength of relationship between a factor and a variable. Example: $F1 \rightarrow v1 = 0.45$. Ranges [-1.0 ... 1.0], same as Pearson's *r*.

Visual model, again



EFA vs Principal Components

Factor analysis is often advantageous over PCA because:
- It's more interpretable; FA aims for clear factor/variable relationships
- FA does this by mathematically *rotating* the components to have clearer loadings (and by omitting non-shared "error" variance)
- FA estimates latent variables in presence of error

Principal components is advantageous when:
- You want an exact reduction of the data regardless of error
- You want to maximize variance explained by the first K components

General Steps for EFA

1. Load and clean the data. Put it on a common scale (e.g., standardize) and address extreme skew.
2. Examine correlation matrix to get a sense of possible factors
3. Determine the number of factors
4. Choose a factor *rotation* model (more in a moment)
5. Fit the model and interpret the resulting factors
6. Repeat 3-5 if unclear, and select the most useful
7. Use factor scores for best estimate of construct/latent variable

Now . . . data!

11 items for simulated product interest and engagement data (PIES), rated on 7 point Likert type scale. We will determine the right number of factors and their variable loadings.

Items:

Paraphrased item	
not important *[reversed]*	never think *[reversed]*
very interested	look for specific features
investigate in depth	some are clearly better
learn about options	others see, form opinion
expresses person	tells about person
match one's image	

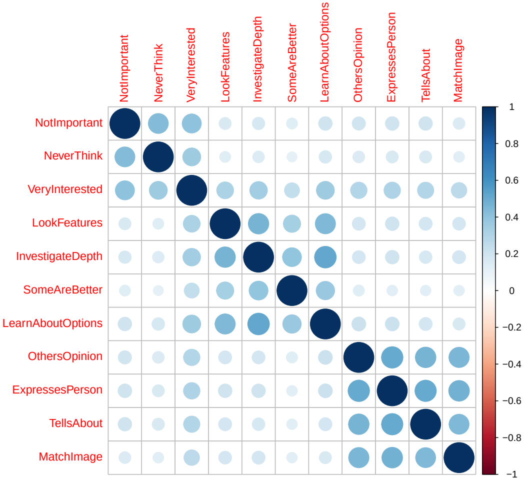
Step 1: Load and clean data

```
pies.data <- read.csv("http://goo.gl/yTOXwJ")
```

```
##                      vars    n mean   sd median trimmed  ma
## NotImportant            1 3600 4.34 1.00      4    4.32 1.4
## NeverThink              2 3600 4.10 1.05      4    4.09 1.4
## VeryInterested          3 3600 4.11 1.02      4    4.10 1.4
## LookFeatures            4 3600 4.04 1.05      4    4.04 1.4
## InvestigateDepth        5 3600 4.00 1.08      4    4.00 1.4
## SomeAreBetter           6 3600 3.92 1.04      4    3.94 1.4
## LearnAboutOptions       7 3600 3.87 1.04      4    3.88 1.4
## OthersOpinion           8 3600 3.90 1.11      4    3.92 1.4
## ExpressesPerson         9 3600 4.02 1.01      4    4.01 1.4
## TellsAbout             10 3600 3.90 1.02      4    3.92 1.4
## MatchImage             11 3600 3.85 1.01      4    3.86 1.4
##                       skew kurtosis   se
## NotImportant         -0.07    -0.01 0.02
```

Step 2: Examine correlation matrix

Correlation plot with clustering is useful (R `corrplot`):



Step 3: Determine number of factors (1)

There is usually not a definitive answer. Choosing number of factors is partially a matter of usefulness.

Generally, look for consensus among:
- Theory: how many do you expect?
- Correlation matrix: how many seem to be there?
- Eigenvalues: how many Factors have Eigenvalue > 1?
- Eigenvalue scree plot: where is the "bend" in extraction?
- Parallel analysis and acceleration [advanced; less used; not covered today]

Step 3: Number of factors: Eigenvalues

In factor analysis, an eigenvalue is the proportion of total shared (i.e., non-error) variance explained by each factor. You might think of it as *volume* in multidimensional space, where each variable adds 1.0 to the volume (thus, `sum(eigenvalues)` = # of variables).

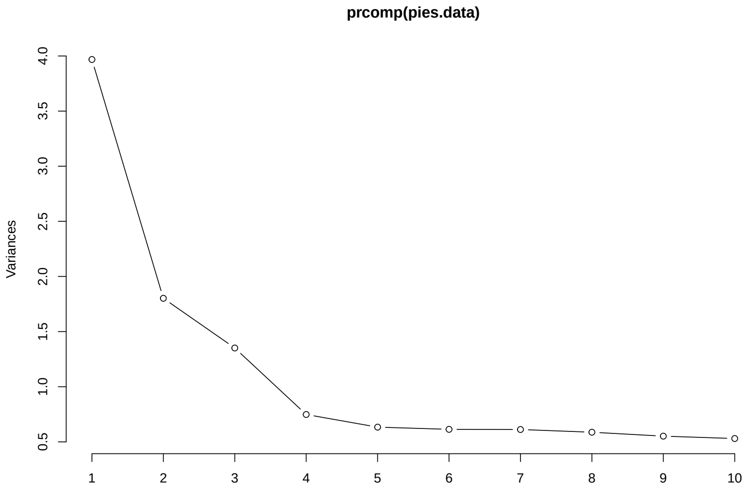
A factor is only useful if it explains more than 1 variable . . . and thus has eigenvalue > 1.0.

```
eigen(cor(pies.data))$values
```

```
##  [1] 3.6606016 1.6422691 1.2749132 0.6880529 0.5800595 0
##  [8] 0.5387749 0.5290039 0.4834441 0.4701335
```

This rule of thumb suggests 3 factors for the present data.

Step 3: Eigenvalue scree plot



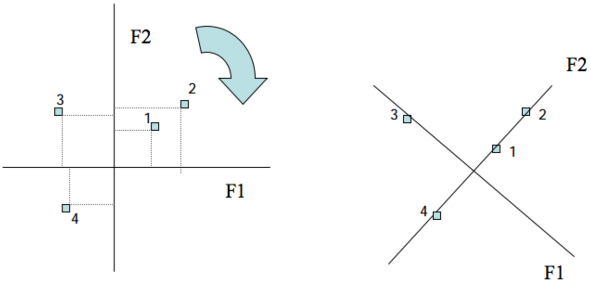
Step 4: Choose a factor rotation model

EFA can be thought of as slicing a pizza. The same material (variance) can be carved up in ways that are mathematically identical, but might be more or less useful for a given situation.

Key decision: do you want the extracted factors to be correlated or not? In FA jargon, *orthogonal* or *oblique*?

By default, EFA looks for orthogonal factors that have $r = 0$ correlation. This maximizes the interpretability, so I recommend using an **orthogonal rotation** in most cases, at least to start. (As a practical matter, it often makes little difference.)

Step 4: Orthogonal Rotation, Conceptually



	Factor 1	Factor 2
x1	0.5	0.5
x2	0.8	0.8
x3	-0.7	0.7
x4	-0.5	-0.5

	Factor 1	Factor 2
x1	0	0.6
x2	0	0.9
x3	-0.9	0
x4	0	-0.9

(*Figure from E. Garrett-Mayer (2006), Statistics in Psychosocial Research. Johns Hopkins. Slides*)

Step 4: Some rotation options

- Default: *varimax*: orthogonal rotation that aims for clear factor/variable structure. Generally recommended.

- Oblique: *oblimin*: finds correlated factors while aiming for interpretability. Recommended if you want an oblique solution.

- Oblique: *promax*: finds correlated factors similarly, but computationally different (good alternative). Recommended alternative if oblimin is not available or has difficulty.

- many others . . . : dozens have been developed. IMO they are useful mostly when you're *very* concerned about psychometrics (e.g., the College Board)

Step 5: Fit the model and interpret the solution

We've decided on 3 factors and orthogonal rotation:

```
library(psych)
pies.fa <- fa(pies.data, nfactors=3, rotate="varimax")
```

Check the eigenvalues (aka sum of squares of loadings)

```
                       MR1  MR2  MR3
SS loadings           1.94 1.83 1.17
Proportion Var        0.18 0.17 0.11
Cumulative Var        0.18 0.34 0.45
Proportion Explained  0.39 0.37 0.24
Cumulative Proportion 0.39 0.76 1.00
```

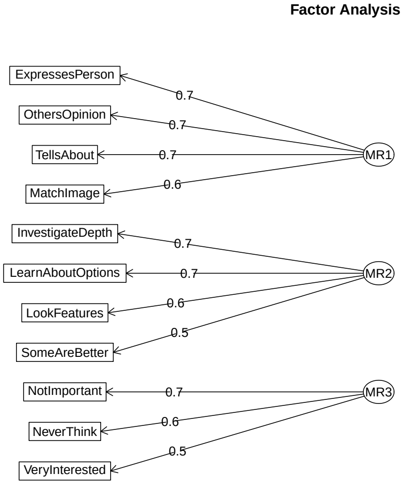
Step 5: Check loadings (L $>$ 0.20 shown)

A rule of thumb is to interpret loadings when $|L| > 0.30$.

```
Loadings:
                  Factor1 Factor2 Factor3
NotImportant                      0.675
NeverThink                        0.614
VeryInterested    0.277   0.362   0.476
LookFeatures              0.608
InvestigateDepth          0.715
SomeAreBetter             0.519
LearnAboutOptions         0.678
OthersOpinion     0.665
ExpressesPerson   0.706
TellsAbout        0.655
MatchImage        0.632
```

Step 5: Visual representation

```
fa.diagram(pies.fa)
```



Step 5: Interpret

We choose names that reflect the factors:

```
                  Image  Feature General
NotImportant                      0.675
NeverThink                        0.614
VeryInterested    0.277   0.362   0.476  # Higher order?
LookFeatures              0.608
InvestigateDepth          0.715
SomeAreBetter             0.519
LearnAboutOptions         0.678
OthersOpinion     0.665
ExpressesPerson   0.706
TellsAbout        0.655
MatchImage        0.632
```

Step 6: Repeat and compare if necessary

We'll omit this today, but we might also want to try some of these:
- More or fewer factors: is the fitted model most interpretable?
- Oblique rotation vs. orthogonal: are the orthogonal factors clear enough?
- Different rotation methods (whether oblique or orthogonal)

And more broadly . . .
- Repeat for samples & different item sets: field other items and cut or keep them according to loadings

Step 7: Use factor scores for respondents

The factor scores are the best estimate for the latent variables for each respondent.

```
fa.scores <- data.frame(pies.fa$scores)
names(fa.scores) <- c("ImageF", "FeatureF", "GeneralF")
head(fa.scores)
```

```
##        ImageF    FeatureF    GeneralF
## 1  0.5101632 -1.23897253  0.79137661
## 2 -0.0710621  0.27993881  0.66318390
## 3 -0.3044523 -0.10334393 -0.87769935
## 4 -0.8640251 -1.10904748  0.42338377
## 5 -0.6915477 -0.08739992 -0.40436752
## 6  1.5312085 -0.38443243 -0.06743736
```

Does rotation work? Comparisons

Compare Rotation: PCA

Difficult to interpret; item variance is spread across components.

```
princomp(pies.data)$loadings[ , 1:3]
```

```
##                      Comp.1       Comp.2     Comp.3
## NotImportant      -0.2382915  0.002701817 -0.5494300
## NeverThink        -0.2246788 -0.010479513 -0.6591112
## VeryInterested    -0.3381471 -0.057999494 -0.2780453
## LookFeatures      -0.3067349 -0.322365785  0.1806022
## InvestigateDepth  -0.3345691 -0.388068434  0.1827942
## SomeAreBetter     -0.2487583 -0.345378333  0.1532197
## LearnAboutOptions -0.3265734 -0.348505290  0.1168231
## OthersOpinion     -0.3504562  0.395258058  0.1629491
## ExpressesPerson   -0.3190419  0.340101339  0.1297465
## TellsAbout        -0.3087084  0.345485125  0.1058713
## MatchImage        -0.2897089  0.331665758  0.1692330
```

Compare Rotation: EFA without rotation

Difficult to interpret; most items load on 2 factors.

```
print(fa(pies.data, nfactors=3, rotate="none")$loadings, cu
```

```
##
## Loadings:
##                     MR1    MR2    MR3
## NotImportant         0.451         0.533
## NeverThink           0.386         0.495
## VeryInterested       0.609
## LookFeatures         0.517  0.334
## InvestigateDepth     0.567  0.428
## SomeAreBetter        0.406  0.324
## LearnAboutOptions    0.571  0.399
## OthersOpinion        0.581 -0.359
## ExpressesPerson      0.610 -0.387
## TellsAbout           0.569 -0.365
```

Compare Rotation: EFA with rotation

Easier to interpret; clear item/factor loadings.

```
print(fa(pies.data, nfactors=3, rotate="varimax")$loadings,
```

```
##
## Loadings:
##                    MR1    MR2    MR3
## NotImportant                     0.675
## NeverThink                       0.613
## VeryInterested            0.362 0.476
## LookFeatures              0.608
## InvestigateDepth          0.715
## SomeAreBetter             0.519
## LearnAboutOptions         0.678
## OthersOpinion      0.665
## ExpressesPerson    0.706
## TellsAbout         0.655
```

Break!

Confirmatory Factor Analysis

CFA primary uses

CFA is a special case of structural equation modeling (SEM), applied to latent variable assessment, usually for surveys and similar data.

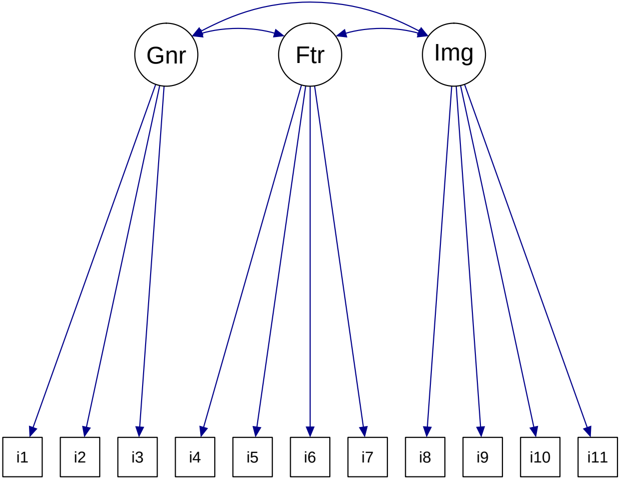
1. Assess the structure of survey scales — do items load where one would hope?
2. Evaluate the fit / appropriateness of a factor model — is a proposed model better than alternatives?
3. Evaluate the weights of items relative to one another and a scale — do they contribute equally?
4. Model other effects such as method effects and hierarchical relationships.

General Steps for CFA

1. Define your hypothesized/favored model with relationships of *latent variables* to *manifest variables*.
2. Define 1 or more *alternative models* that are reasonable, but which you believe are inferior.
3. Fit the models to your data.
4. Determine whether your model is good enough (fit indices, paths)
5. Determine whether your model is better than the alternative
6. Intepret your model (Optional: do a little dance. You deserve it!)

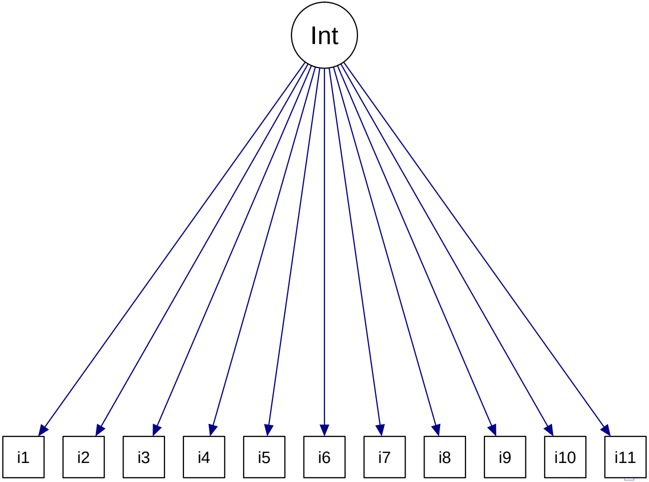
Target CFA Model for PIES

We'll define a **3-factor model** with potentially correlated factors.



Comparative model for PIES

Compare a **1-factor model** where all variables load on one *interest* factor. Our 3-factor model must fit better than this to be of interest!



Model fit: Fit Measures

Global fit indices
Example: Comparative Fit Index (CFI). Attempts to assess "absolute" fit vs. the data. Not very good measures, but set a minimum bar: want fit > 0.90.

Approximation error and residuals
Example: Standardized Root Mean Square Residual (SRMR). Difference between the data's covariance matrix and the fitted model's matrix. Want SRMR < 0.08. For Root Mean Square Error of Approximation, want Lower-CI(RMSEA) < 0.05.

Information Criteria
Example: Akaike Information Criterion (AIC). Assesses the model's fit vs. the observed data. No absolute interpretation, but lower is better. Difference of 10 or more is large.

R code to fit the 3-factor model

It is very simple to define and fit a CFA in R!

```
library(lavaan)

piesModel3 <- " General =~ i1 + i2 + i3
                Feature =~ i4 + i5 + i6  + i7
                Image   =~ i8 + i9 + i10 + i11 "

pies.fit3 <- cfa(piesModel3, data=pies.data)

summary(pies.fit3, fit.measures=TRUE)
```

Model fit: 1-Factor model

```
> piesModel1 <- " Interest =~ i1 + i2 + i3 + i4 + i5 + i6
+                             i8 + i9 + i10 + i11 "
> pies.fit1 <- cfa(piesModel1, data=pies.data)
> summary(pies.fit1, fit.measures=TRUE)

  Comparative Fit Index (CFI)          0.672   # Bad

  Akaike (AIC)                    108812.709   # Much higher
  Bayesian (BIC)                  108948.860

  RMSEA                                0.143   # Bad
  90 Percent Confidence Interval  0.139  0.147
  P-value RMSEA <= 0.05                0.000   # Bad
  SRMR                                 0.102   # Bad
```

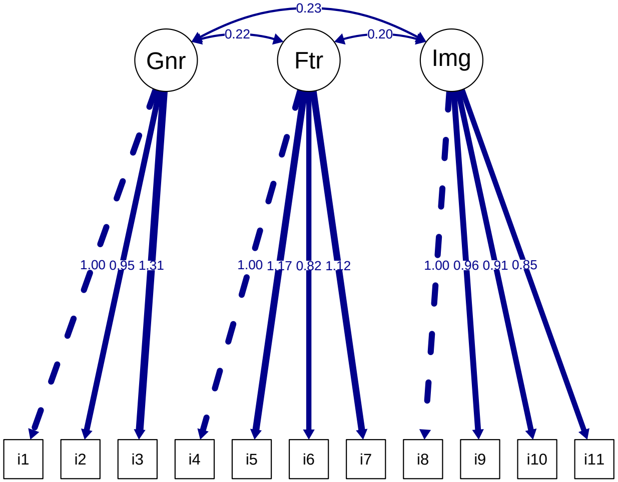
Model fit: 3-Factor model

```
> piesModel3 <- " General =~ i1 + i2 + i3
+                 Feature =~ i4 + i5 + i6  + i7
+                 Image   =~ i8 + i9 + i10 + i11 "
> pies.fit3 <- cfa(piesModel3, data=pies.data)
> summary(pies.fit3, fit.measures=TRUE)

  Comparative Fit Index (CFI)         0.975   # Excellent

  Akaike (AIC)                   105821.776   # Much lower
  Bayesian (BIC)                 105976.494

  RMSEA                               0.041   # Excellent
  90 Percent Confidence Interval  0.036  0.045
  P-value RMSEA <= 0.05               1.000   # Good
  SRMR                                0.030   # Excellent
```

Model Paths

Latent Variables	Estimate	Std.Err	Z-value	P(>|z|)
General =~				
i1	1.000			
i2	0.948	0.042	22.415	0.000
i3	1.305	0.052	25.268	0.000
Feature =~				
i4	1.000			
i5	1.168	0.037	31.168	0.000
i6	0.822	0.033	25.211	0.000
i7	1.119	0.036	31.022	0.000
Image =~				
i8	1.000			
i9	0.963	0.028	34.657	0.000
i10	0.908	0.027	33.146	0.000
i11	0.850	0.027	31.786	0.000

Visualize it



A few other points on CFA

Fixing paths
To make a model identifiable, one path must be fixed between a factor and a variable. This makes paths interpretable relative to that variable. Standardizing predictors is important so they're comparable!

Modeling Factor Correlations
You might specify some factor correlations as low (e.g., 0.10) or high (e.g., 0.50). This is easy to do in R; differs in other packages.

Hierarchical Models
You can model higher-order factors, such as overall "product interest". CFA allows latent variables associated with other latent variables ... easy, but too complex for today! (See Chapter 10 in Chapman & Feit for an example.)

Factor Loadings vs. Path Coefficients

By default, EFA and CFA report factor::variable coefficients on different scales.

EFA Factor Loadings

EFA Loadings are on Pearson's *r* scale, i.e., they are correlation coefficients between the factor and the variable.

CFA Path Coefficients

CFA paths are in the variables' own scale(s), fixed to 1.0 for one variable per factor. If you standardize (Z-score, i.e. $\frac{X-\bar{X}}{sd(X)}$) variables and fix factor variances to 1.0, they will be on the *r* scale. (See a CFA reference; this is an option in many CFA packages.) **Note**: Having paths on original scale is often useful in survey analysis, because it is easy to compare to the scale itself.

Finally

The Main Points

1. If you use scales with multiple items, check them with EFA & CFA! Don't just assume that your scales are correct, or that items load where you expect.
2. If you have multiple item scores – such as items that add up to a "satisfaction" score – consider using factor scores instead.
3. If you propose a complex model, prove with CFA that it's better than alternatives.
4. This area has a lot of jargon but is not intrinsically difficult . . . and is much better than ignoring it and hoping for the best! SPSS, R, SAS, and Stata all have excellent factor analysis tools.

Workflow for scale development

1. Identify factors of possible interest
2. Write many items for those factors and field a survey
3. Use EFA to identify whether the factors hold up, and which items load on them
4. Repeat 1-3 until you believe you have reliable factors and good items
5. Use CFA to demonstrate that the factors and items hold up in a new sample

Example: PIES scale development paper, Chapman et al, 2009

Learning more: Books

1. Chapman & Feit (2015), *R for Marketing Research and Analytics*. Chapters 8 and 10 present EFA and CFA in marketing contexts, with detailed R code.
2. Pett, Lackey, & Sullivan (2003), *Making Sense of Factor Analysis*. A practical guide to EFA with emphasis on (healthcare) survey development.
3. Brown (2015), *Confirmatory Factor Analysis for Applied Research*. An excellent & practical text on CFA for the social sciences.
4. Kline (2015), *Principles and Practice of Structural Equation Modeling*. The definitive guide for social science usage of CFA and SEM.
5. DeVellis (2011), *Scale Development*. A very practical and readable guide to building good survey scales.

Software Notes: R Packages

1. `corrplot` is helpful to explore correlation matrices before attempting factor analysis.
2. `psych` has many options for exploratory factor analysis and other psychometric (survey/test) applications.
3. `lavaan` is an easy to use SEM/CFA package. It supports intuitive definition of models with simple equation notation. Highly recommended.
4. `semPlot` makes basic SEM/CFA path diagrams as shown here.
5. `DiagrammeR` makes publication quality path diagrams. It requires manual specification — tedious but not difficult — for EFA/CFA/SEM diagrams.
6. `OpenMX` provides advanced options for SEM such as mixed effects models.

Thank you!

Questions?

Contact: `cchapman@google.com`